Means and Variances (Q 1.5)

1. What happens we subtract the means for each of the derived samples? Why?

2. And the variances? Why? What is a real world example?
# Reviewing Distributions

<table>
<thead>
<tr>
<th></th>
<th><strong>$Z$ Distribution</strong></th>
<th><strong>$t$ Distribution</strong></th>
<th><strong>$\chi^2$ Distribution</strong></th>
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<tr>
<td><strong>When to use</strong></td>
<td>Used when the true population variance (either of individuals or sample means) is known</td>
<td>Used when the true population variance is unknown and is estimated by a sample variance (must pay price for uncertainty)</td>
<td>1. Defined as the SS of standard normal variables ($Z_i$). 2. Since the $Z$ variable is squared, $\chi^2$ has only positive values</td>
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<td><strong>Associated Equations</strong></td>
<td>$Z = \frac{Y_i - \mu}{\sigma}$</td>
<td>$Z = \frac{\bar{Y} - \mu}{\sigma_{\bar{Y}}}$</td>
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</table>
$t_{P,df} = \frac{Y_i - \mu}{s_n}$  
$t_{P,df} = \frac{\bar{Y} - \mu}{s_{\bar{Y}}_n}$ |
| **Characteristics**  | 1. $Z(Y)$ is the height of the curve at a given observed value $Y$  
2. Location and shape of a normal probability density function determined by two parameters, $\mu$ and $\sigma^2$ | 1. Tends toward $Z$ as sample size increases ($t \rightarrow Z$) as ($df \rightarrow \infty$)  
2. Usually $df << \infty$; so the $t$ distributions are usually flatter and more dispersed than a standard normal distribution  
3. One or two tailed probabilities | 1. If only one $Z$ distribution is involved in the sum, the $\chi^2$ distribution is said to have 1 df  
2. For each number of df, there is a $\chi^2$ distribution  
3. The two tails of the $Z$ distribution are brought together in a single tail of $\chi^2$, meaning $\chi^2_{1, \alpha} = Z^2_{\alpha/2}$ |
Example #1

From a normally distributed population of finches with mean weight ($\mu$) = 17.2 g and variance ($\sigma^2$) = 36 g$^2$, what is the probability of randomly selecting an individual finch weighing more than 22 g?

Probability of selecting an individual of a certain value at random

\[ Z = \frac{Y_i - \mu}{\sigma} \]

A. $Z = \frac{Y_i - \mu}{\sigma}$

B. $Z = \frac{\bar{Y} - \mu}{\sigma_{\bar{Y}}}$

C. $t_{P, df} = \frac{Y_i - \mu}{S_n}$

D. $t_{P, df} = \frac{\bar{Y} - \mu}{S_{\bar{Y}_n}}$
Example #2

From a population of finches with mean weight \( \mu = 17.2 \text{ g} \) and sample variance \( s^2 = 36 \text{ g}^2 \), what is the probability of of randomly selecting an individual finch weighing more than 22 g from a sample of 20 finches?

Probability of selecting an individual of a certain value at random from a sample with an unknown population variance

\[
\begin{align*}
A. & \quad Z = \frac{Y_i - \mu}{\sigma} \\
B. & \quad Z = \frac{\bar{Y} - \mu}{\sigma_{\bar{Y}}} \\
C. & \quad t_{P,df} = \frac{Y_i - \mu}{s_n} \\
D. & \quad t_{P,df} = \frac{\bar{Y} - \mu}{s_{\bar{Y}_n}}
\end{align*}
\]
From a population of finches with mean weight ($\mu$) = 17.2 g and sample variance ($s^2$) = 36 g$^2$, what is the probability of a sample of 20 finches with an average weight of more than 22 g?

Probability of selecting a sample of a certain value at random with an unknown population variance

Example #3

$$Z = \frac{Y_i - \mu}{\sigma} \quad \text{B.} \quad Z = \frac{\bar{Y} - \mu}{\sigma_{\bar{Y}}} \quad \text{C.} \quad t_{P,df} = \frac{Y_i - \mu}{s_n} \quad \text{D.} \quad t_{P,df} = \frac{\bar{Y} - \mu}{s_{\bar{Y}_n}}$$
Example #4

From a normally distributed population of finches with mean weight ($\mu$) = 17.2 g and variance ($\sigma^2$) = 36 g$^2$, what is the probability of randomly selecting a sample of 20 finches with an average weight of more than 22 g?

Probability of selecting a sample of a certain value at random

A. $Z = \frac{Y_i - \mu}{\sigma}$  
B. $Z = \frac{\bar{Y} - \mu}{\sigma_{\bar{Y}}}$  

C. $t_{P,df} = \frac{Y_i - \mu}{S_n}$  
D. $t_{P,df} = \frac{\bar{Y} - \mu}{S_{\bar{Y}}n}$
Example #5

The Acme Battery Company has developed a new cell phone battery. On average, the battery lasts 60 minutes on a single charge. The standard deviation is 4 minutes. Suppose the manufacturing department runs a quality control test. They randomly select 7 batteries. The standard deviation of the selected batteries is 6 minutes. What is the probability that the standard deviation in the new test would be greater than 6 minutes?

\[ X^2 = \frac{(n - 1) * s^2}{\sigma^2} \]

\[ X^2 = \frac{(7 - 1) * 6^2}{4^2} = 13.5 \]

To find the cumulative probability that a chi-square statistic falls between 0 and 13.5

The cumulative probability: 0.96

Probability that a SD would be less than or equal to 6 minutes is 0.96

The probability that the standard deviation would be greater than 6 minutes is 1 - 0.96 or .04.
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