**ANCOVA for a 3x2 Factorial arranged as an RCBD**

**Data**
```
ABFact;
   Input Block A B X Y @@;
   Z = Y – Beta * (X - Xmean);
Cards;
```

**Proc GLM:**
```
   Class Block A B;
   Model X Y = Block A B A*B;
```

**Proc GLM:**
```
   Class Block A B;
   Model Y = Block A B A*B X / solution;
   LSMeans A;
   LSMeans B;
   Contrast 'A linear' A -1 0 1;
Run;
Quit;
```

1. To test normality of residuals

**Proc GLM:**
```
   ANOVA on Z
   Class Block A B;
   Model Z = Block A B A*B;
   Output out = ABFactPRz p = Pred r = Res;
```

**Proc Univariate**
```
   Data = ABFactPRz normal;
   Var Res;
```

2. To test homogeneity of variances

3. To test homogeneity of slopes

**Proc GLM:**
```
   Class Block TRT;
   Model Z = TRT;
   Means TRT / hovtest = Levene;
```

Create a Treatment ID (TRT) with 6 levels, one for each A-B combination
4. To partition the A*B interaction

Model Y = Block TRT X;
LSMeans TRT;
Contrast 'A Linear' TRT -1 0 1 -1 0 1;
Contrast 'A Quadratic' TRT 1 -2 1 1 -2 1;
Contrast 'B' TRT 1 1 1 -1 -1 -1;
Contrast 'A Lin x B' TRT -1 0 1 1 0 -1;
Contrast 'A Quad x B' TRT 1 -2 1 -1 2 -1;

5. To perform a trend analysis of unequally-spaced treatment levels (Factor A)

First, partition the sum of squares associated with A:

Class Block B; ← A is removed from the Class statement
Model Y = Block B X A A*A;

Then, test the two components of A by hand using the appropriate error term:

F (linear A) = MS(A) / MSE from full model
F (quad A) = MS(A) / MSE from full model

6. To analyze a mixed model (Factor A random, Factor B fixed)

Model Y = Block A B A*B X;
Random Block A A*B / test;
Test h = A e = A*B;
Contrast 'A Linear' A -1 0 1 / e = A*B;

<table>
<thead>
<tr>
<th>Source</th>
<th>Type III Expected Mean Square</th>
</tr>
</thead>
<tbody>
<tr>
<td>Block</td>
<td>Var(Error) + 6 Var(Block)</td>
</tr>
<tr>
<td>A</td>
<td>Var(Error) + 4 Var(A*B) + 8 Var(A)</td>
</tr>
<tr>
<td>B</td>
<td>Var(Error) + 4 Var(A*B) + Q(B)</td>
</tr>
<tr>
<td>A*B</td>
<td>Var(Error) + 4 Var(A*B)</td>
</tr>
</tbody>
</table>
Mixed Model: Blocks nested within Random Locations

Five varieties of barley are tested in three locations selected at random across the Sacramento Valley. In each location, the experiment is organized as an RCBD with four blocks. The intention is to:

a. Characterize the variability across the valley.

b. Characterize the stability of variety performance across the valley.

c. Recommend a variety(ies) for the valley.

<table>
<thead>
<tr>
<th>Class Variable</th>
<th>Block or Treatment</th>
<th>Number of Levels</th>
<th>Fixed or Random</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>↓</td>
<td>n</td>
<td></td>
</tr>
</tbody>
</table>

Proc GLM:
Class Location Block Variety;
Model Yield = Location Block(Location) Variety Location*Variety;
Random Location Block(Location) Location*Variety / test;

Source | Type III Expected Mean Square
--- | ---
Location | $\sigma^2_{\text{Error}} + b\sigma^2_{\text{LOC*VAR}} + \nu\sigma^2_{\text{BLOCK(LOC)}} + b\nu\sigma^2_{\text{LOC}}$
Block(Location) | $\sigma^2_{\text{Error}} + \nu\sigma^2_{\text{BLOCK(LOC)}}$
Variety | $\sigma^2_{\text{Error}} + b\sigma^2_{\text{LOC*VAR}} + (b*\nu)\sum_{i=1}^{v} v_i^2$
Location*Variety | $\sigma^2_{\text{Error}} + b\sigma^2_{\text{LOC*VAR}}$

Proc VarComp Method = Type1;
Class Location Block Variety;
Model Y = Variety Location Block(Location) Location*Variety / Fixed = 1;

<table>
<thead>
<tr>
<th>Variance Component</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma^2_{\text{Location}}$</td>
<td>60.45326</td>
</tr>
<tr>
<td>$\sigma^2_{\text{Location*Variety}}$</td>
<td>12.52485</td>
</tr>
<tr>
<td>$\sigma^2_{\text{Block(Location)}}$</td>
<td>15.16845</td>
</tr>
<tr>
<td>$\sigma^2_{\text{Error}}$</td>
<td>150.87298</td>
</tr>
</tbody>
</table>
Three-way factorials with one split (NOT a split-split-plot)

Example 1: RCBD with... Main plot = Factorial treatment structure (A*B) Subplot = Factor C

<table>
<thead>
<tr>
<th></th>
<th>a2-b1</th>
<th>a1-b2</th>
<th>a1-b1</th>
<th>a2-b2</th>
</tr>
</thead>
<tbody>
<tr>
<td>BLOCK 1</td>
<td>c3</td>
<td>c1</td>
<td>c2</td>
<td>c2</td>
</tr>
<tr>
<td></td>
<td>c2</td>
<td>c2</td>
<td>c1</td>
<td>c3</td>
</tr>
<tr>
<td></td>
<td>c1</td>
<td>c3</td>
<td>c3</td>
<td>c1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>a1-b1</th>
<th>a2-b2</th>
<th>a2-b1</th>
<th>a1-b2</th>
</tr>
</thead>
<tbody>
<tr>
<td>BLOCK 2</td>
<td>c1</td>
<td>c2</td>
<td>c1</td>
<td>c3</td>
</tr>
<tr>
<td></td>
<td>c3</td>
<td>c1</td>
<td>c2</td>
<td>c2</td>
</tr>
<tr>
<td></td>
<td>c2</td>
<td>c3</td>
<td>c3</td>
<td>c1</td>
</tr>
</tbody>
</table>

Proc GLM:

Class Block A B C;
Test h = A e = Block*A*B;
Test h = B e = Block*A*B;
Test h = A*B e = Block*A*B;
Means A / Tukey e = Block*A*B;
Contrast 'B Linear' B 1 0 -1 / e = Block*A*B; ← If B had three levels, for example

Alternatively, open up the A*B factorial, expressing the combinations as levels of some single factor TRT:

Proc GLM:

Class Block TRT C;
Model Y = Block TRT Block*TRT C C*TRT;
Test h = TRT e = Block*TRT;
Means TRT / Tukey e = Block*TRT;
Contrast 'B Linear' B 1 0 -1 1 0 -1 / e = Block*TRT;
Example 2: RCBD with...
Main plot = Factor A
Subplot = Factorial treatment structure (B*C)

<table>
<thead>
<tr>
<th>BLOCK 1</th>
<th>a2</th>
<th>a1</th>
</tr>
</thead>
<tbody>
<tr>
<td>b1-c3</td>
<td>b2-c1</td>
<td>b2-c2</td>
</tr>
<tr>
<td>b2-c2</td>
<td>b1-c2</td>
<td>b2-c1</td>
</tr>
<tr>
<td>b1-c1</td>
<td>b2-c3</td>
<td>b2-c3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>BLOCK 2</th>
<th>a1</th>
<th>a2</th>
</tr>
</thead>
<tbody>
<tr>
<td>b1-c1</td>
<td>b1-c2</td>
<td>b2-c1</td>
</tr>
<tr>
<td>b2-c3</td>
<td>b2-c1</td>
<td>b1-c2</td>
</tr>
<tr>
<td>b2-c2</td>
<td>b1-c3</td>
<td>b2-c3</td>
</tr>
</tbody>
</table>

**Proc GLM:**
- **Class** Block A B C;
- **Model** Y = Block A Block*A
  - B C A*C B*C A*B*C;
  - **Test** h = A e = Block*A;
- **Means** A / Tukey e = Block*A;
- **Contrast** 'A Linear' A 1 0 -1 / e = Block*A; ← If A had three levels, for example

Alternatively, open up the B*C factorial, expressing the combinations as levels of some single factor TRT:

**Proc GLM:**
- **Class** Block A TRT;
- **Model** Y = Block A Block*A
  - TRT A*TRT;
- **Test** h = A e = Block*A;
A 4x3x3 factorial experiment, organized as an RCBD

**Proc GLM:**

```r
Class Block A B C;
Model Y = Block A|B|C;
```

If A*B and A*C are significant, but B*C is not significant:

**Proc Sort:**

```r
By A;
```

**Proc GLM:**

```r
Class Block B C;
Model Y = Block B|C;
Means B / Tukey;
Means C / Tukey;
Contrast 'Linear B' B 1 0 -1;
By A;
```

**Proc GLM:**

```r
Class Block A;
Model Y = Block A;
Means A / Tukey;
Contrast 'Linear A' A -3 1 1 3;
By B C;
```

If A*B is significant, but A*C and B*C are not significant:

It is valid to analyze the main effect of C:

- e.g.  
  ```r
  Contrast 'Linear C' C 1 0 -1;
  Means C / Tukey;
  ```

But for A and B:

1. Describe the effect of A within each level of B (across all C levels)
2. Describe the effect of B within each level of A (across all C levels)

**Proc Sort:**

```r
By B;
```

**Proc GLM:**

```r
Class Block A C;
Model Y = Block A|C;
By B;
```

**Proc GLM:**

```r
Class Block B C;
Model Y = Block B|C;
By A;
```

If all three 2-way interactions are significant:

1. Describe the effect of A within each combination of B and C
2. Describe the effect of B within each combination of A and C
3. Describe the effect of C within each combination of A and B
Calculating Power

\[ F_{\alpha, df_{NUM}, df_{DEN}} = \frac{MS(NUM)}{MS(DEN)} \]

\[ \phi = \sqrt{\frac{r}{MSE} \sum_{i} \frac{\tau_i^2}{t}} \rightarrow v_1 = df_{NUM}, v_2 = df_{DEN} \]

Power to detect differences among treatments in…
RCBD with 1 rep per cell
RCBD with 2 reps per cell

Power to detect differences among treatments in…
AxB split-plot (RCBD)
AxB strip-plot (RCBD)

AxB split-plot (RCBD), where B is a random factor:

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<tr>
<td>Block</td>
<td>Var(Error) + 4 Var(Block*A) + 12 Var(Block)</td>
</tr>
<tr>
<td>A</td>
<td>Var(Error) + 4 Var(A<em>B) + 4 Var(Block</em>A) + Q(A)</td>
</tr>
<tr>
<td>Block*A</td>
<td>Var(Error) + 4 Var(Block*A)</td>
</tr>
<tr>
<td>B</td>
<td>Var(Error) + 4 Var(A*B) + 12 Var(B)</td>
</tr>
<tr>
<td>A*B</td>
<td>Var(Error) + 4 Var(A*B)</td>
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</tbody>
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Language

_Homogeneity = uniformity_

**Homogeneity of variances across treatment levels:** The variance among experimental units within a treatment level is uniform across all treatment levels. (Levene's Test is NS)

**Homogeneity of treatment effects across blocks:** The effect of treatment on Y is uniform across all blocks. (Block*Trt is NS)

**Homogeneity of the effect of A across all levels of B:** The effect of A does not depend on the level of B. The effect of A is uniform across all levels of B. (A*B is NS)
Blocks vs. Treatments

Is the purpose of the factor to control error?
- YES → Block
- NO → Treatment

Are you interested in characterizing the factor's effect (fixed) or its variability (random)?
- NO → Block
- YES → Treatment

Are you interested in characterizing its interactions with other factors?
- NO → Block
- YES → Treatment

Blocks vs. Covariable

1. Are the levels of the factor discrete or continuous?
   - Discrete → Blocks
   - Continuous → either

2. Do you have individual values for each experimental unit in the study?
   - NO → Blocks
   - YES → Covariable

Time as Treatment (Factor vs. Subplot vs. Repeated Measure)

Scenario 1: Raspberries harvested, placed in dishes, and each dish measured daily.

Scenario 2: Raspberries harvested, placed in dishes, and each dish measured once, on its randomly assigned day.

Scenario 3: Raspberries harvested by variety, placed in dishes, and each dish measured daily.

Scenario 4: Raspberries harvested by variety, placed in dishes, and each dish measured once, on its randomly assigned day.

Scenario 5: Raspberries harvested by variety and maturity, placed in dishes, and each dish measured daily.

Etc.