

Topic 9. Factorial Experiments [ST&D Chapter 15]

9. 1. Introduction

- In earlier times factors were studied one at a time, with separate experiments devoted to each factor.
- In the factorial approach, the investigator compares **all** treatments that can be formed by combining the **levels** of the different factors.
- Factorial experimentation is **highly efficient**, because every observation supplies information about all the factors included in the experiment.
- Factorial experimentation is a systematic method of investigating the relationships and **interactions** between the effects of different factors.

9. 2. Terminology

- The individual treatments are called **factors**.
- The treatment levels within a factor are called **levels**.
- If factor A has 3 levels and factor B has 5 then it is a 3 x 5 factorial experiment.

9. 3. Example of a 2x2 factorial

An example of an experiment involving two factors is the application of two nitrogen levels, N_0 and N_1 , and two phosphorous levels, P_0 and P_1 to a crop, with yield (lb/a) as the measured variable. The results are shown here:

Factor			A = N level		
	Level	a1 = N_0	a2 = N_1	Mean	$a_2 - a_1$
B=	b1 = P_0	40.9	47.8	44.4	6.9 (<i>se</i> A,b1)
P level	b2 = P_1	42.4	50.2	46.3	7.8 (<i>se</i> A,b2)
	Mean	41.6	49.0	45.3	7.4 (<i>me</i> A)
	$b_2 - b_1$	1.5 (<i>se</i> B,a1)	2.4 (<i>se</i> B,a2)	1.9 (<i>me</i> B)	

- The differences $a_2 - a_1$ at b_1 and $a_2 - a_1$ at b_2 are the **simple effects**, denoted (*se* A) and (*se* B). For example, 6.9 lb/a is the simple effect of N on yield at P_0 .
- The averages of the simple effects are the **main effects**, denoted (*me* A) and (*me* B) or simply (A) and (B). For example, 7.4 lb/a is the *main effect* (*me*) of N on yield.

9. 4. Interaction

- **Interaction** measures the failure of one effect to be the same for each level of other factors.
- **Interaction** is a common and a fundamental scientific idea.

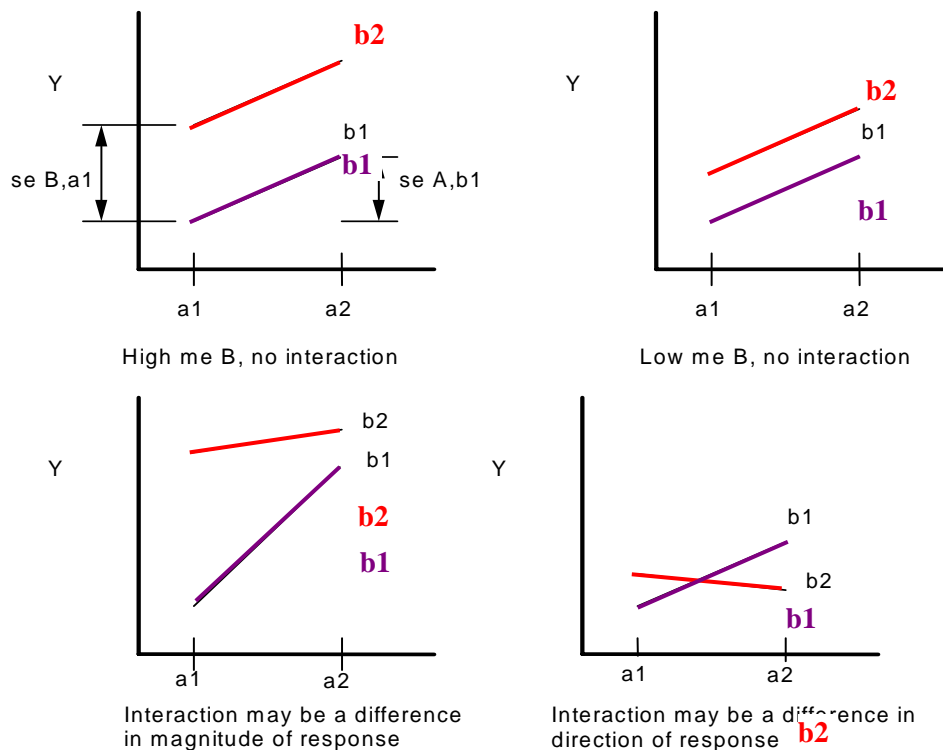
- One of the main objectives of factorial experiments is to study the interactions between factors.
- The SS interaction measures the departure of the group means from the values expected on the basis of additive combinations of the row and column means.

Synergism: the combination of levels of 2 factors *enhance* the effect.

Interference: combination of levels of 2 factors *inhibit* each other's effect.

Synergism and interference both tend to magnify the interaction SS.

These differences between simple effects of two factors or **first-order interactions** (AxB) can be visualized in the following **interaction graphs**.



If the simple effects of Factor A are the same for all levels of Factor B the two factors are said to be **independent** or **additive** (parallel lines).

9. 5. 1. Reasons for doing factorial experiments

- To investigate the **interactions** of factors. Single factor experiments provide a disorderly and incomplete picture.
- In **exploratory work** for quick determination of which factors are independent and can therefore be more fully analyzed in separate experiments.
- To lead to recommendations that must **extend** over a wide **range of conditions**.

9. 5. 2. Disadvantages of factorial design

- **Large number of combinations** required to study several factors at several levels and need a large sized experiment: 7 factors at 3 levels requires 2187 combinations.
- Large number of factors complicate the **interpretation** of high order interactions

9. 6. Differences between nested and factorial experiments

Consider a **factorial experiment** in which growth of leaf discs was measured in tissue culture with 5 different sugars at two different pH levels.

Compare it with a **nested design** in which each sugar solution is prepared twice, so there are two batches of sugar made up for each treatment

Two way factorial ANOVA						Nested ANOVA										
	S1	S2	S3	S4	S5	S1		S2		S3		S4		S5		Sugar batches
pH 1	*	*	*	*	*	1	2	1	2	1	2	1	2	1	2	
	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	
	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	
	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	
	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	
pH 2	*	*	*	*	*											
	*	*	*	*	*											
	*	*	*	*	*											
	*	*	*	*	*											
	*	*	*	*	*											

The **factorial analysis** implies that the exp. units assigned to pH1 have something in **common** that differentiate them from the e.u. assigned to pH2.

In the **nested design** batch 1 in S1 is equally related to batches 1 and 2 in S2.

9. 7. The two-way factorial analysis

9. 7. 1. The linear model for two-way factorial experiments

The linear model for a two-factor analysis is

$$Y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk}$$

α_i represents the main effect of factor A i, $i = 1, \dots, a$,

β_j represents the main effect of factor B, $j = 1, \dots, b$,

$(\alpha\beta)_{ij}$ represents the interaction of factor A level i with factor B level

ε_{ijk} is the error associated with rep k of factor level ij, $k = 1, \dots, r$.

The H_0 for a 2 factor experiment are $\alpha_i = 0$, $\beta_j = 0$, and $(\alpha\beta)_{ij} = 0$.

9. 7. 2. ANOVA for a two-way factorial design

In ANOVA for factorial experiments the SS of *treatments* is partitioned into components for each factor and each interaction.

$$SS = SSA + SSB + SSAB + SSE.$$

This partition is valid even when the overall F test of no differences among treatments is not significant. A significant response to A might well be lost in an overall test of significance.

The ANOVA table for the CRD factorial design is

Source	df	SS	MS	F
Factor A	a - 1	SSA	MSA	MSA/MSE
Factor B	b - 1	SSB	MSB	MSB/MSE
AxB	(a - 1)(b - 1)	SSAB	MSAB	MSAB/MSE
Error	ab(r - 1)	SSE	MSE	
Total	rab - 1	SS		

- **No significant interaction**: multiple comparisons can be performed on the **main effect means**.
- **Significant interaction**: go back to the means and analyze **simple effects**. Compare the means of one factor separately for each level of the other factor

Difference between RCBD and FACTORIAL

The RCBD differs from a true factorial design in the *objective*.

RCBD: We are not interested in the effect of the blocking factor or in the *interaction* between the block and the factor. The main interest is to separate an additional source of variation and we assume no interactions.

Relationship between factorial experiments and experimental design

Experimental design is concerned with the assignment of treatments to experimental units,

A **factorial experiment** is concerned with the structure of treatments. The factorial structure may be placed into any experimental design.

Example of a 2x4 Factorial experiment replicated in different designs

Factor **A** at 2 levels (1, 2)

Factor **B** at 4 levels (1, 2, 3, 4)

Eight different combinations of both factors: 11 12 13 14 21 22 23 24

CRD with 3 replicates of the factorial experiment

24 23 13 23 24 14 13 23 11 24 12 14 22 13 12 21 21 11 22 12 11 22 21 14

RCBD with 3 blocks

13 12 21 23 11 24 14 22

12 11 24 23 13 22 21 14

24 14 22 21 11 13 23 12

8 x 8 Latin Square

24	11	22	12	13	14	23	21
21	23	13	14	22	12	11	24
12	14	24	11	23	21	22	13
13	22	21	24	11	23	14	12
23	12	11	13	21	22	24	14
14	24	23	22	12	13	21	11
11	21	12	23	14	24	13	22
22	13	14	21	24	11	12	23

9. 7. 4. Example of a 2 x 3 factorial organized in a RCBD with no significant interactions (ST&D Table 15.3 p 391)

Square root of the number of quack-grass shoots per square foot after spraying with maleic hydrazide.

Treatments are maleic hydrazide applications rates (**R**) of 0, 4, and 8 lb/acre, and days delay in cultivation after spray (**D**, 3 or 10 days)

D	R	Block 1	Block 2	Block 3	Block 4	Total
3	0	15.7	14.6	16.5	14.7	61.5
	4	9.8	14.6	11.9	12.4	48.7
	8	7.9	10.3	9.7	9.6	37.5
10	0	18.0	17.4	15.1	14.4	64.9
	4	13.6	10.6	11.8	13.3	49.3
	8	8.8	8.2	11.3	11.2	39.5
Totals		73.8	75.7	76.3	75.6	301.4

Randomization?

SAS Program

```

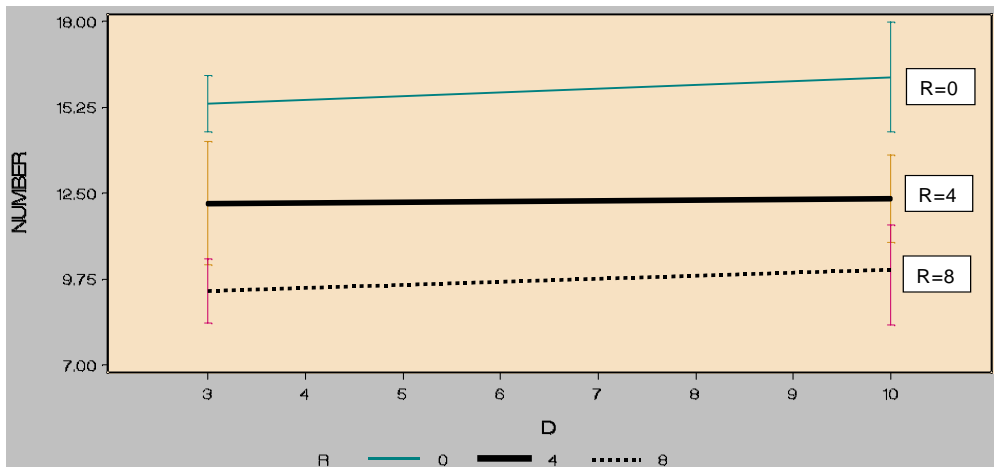
data STDp391;
input D R block number @@;
cards;
3 0 1 15.7 3 4 1 9.8 3 8 1 7.9
3 0 2 14.6 3 4 2 14.6 3 8 2 10.3
3 0 3 16.5 3 4 3 11.9 3 8 3 9.7
3 0 4 14.7 3 4 4 12.4 3 8 4 9.6
10 0 1 18.0 10 4 1 13.6 10 8 1 8.8
10 0 2 17.4 10 4 2 10.6 10 8 2 8.2
10 0 3 15.1 10 4 3 11.8 10 8 3 11.3
10 0 4 14.4 10 4 4 13.3 10 8 4 11.2
proc GLM;
class D R block;
model number= block D R D*R;
means D|R;

```

**If D*R not included...
Which interactions are missing?**

Dependent Variable: NUMBER

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	8	156.235	19.529	7.44	0.0005
Error	15	39.383	2.626		
Corrected Total	23	195.618			
BLOCK	3	0.582	0.194	0.07	0.9731
D	1	1.500	1.500	0.57	0.4614
R	2	153.663	76.832	29.26	0.0001
D*R	2	0.490	0.245	0.09	0.9114



Plot of the dependent means for the two-way effects

Parallel lines indicate absence of interaction: the differences among “R” doses are the same for the different “D” levels.

If no interactions are present the next step is the analysis of the main effects.

Multiple comparisons can be performed using the means of the main effects using CONTRAST or the multiple comparison tests:

```
means D|R / lsd;
```

or

```
contrast 'R lineal'      R -1  0  1;
contrast 'R quadratic'  R  1 -2  1;
```

Partition of the interaction sum of squares

Create a variable whose values are the combinations of levels of the different factors

D3 R0 = DR1,

D3 R4 = DR2,

D3 R8 = DR3,

D10 R0= DR4,

D10 R4= DR5,

D10 R8= DR6.

Analyze TRT means as if TRT were a **one-way classification** of the data.

```

data STDp391c;
input DR D R block number @@;
cards;
1 3 0 1 15.7 2 3 4 1 9.8 3 3 8 1 7.9
1 3 0 2 14.6 2 3 4 2 14.6 3 3 8 2 10.3
1 3 0 3 16.5 2 3 4 3 11.9 3 3 8 3 9.7
1 3 0 4 14.7 2 3 4 4 12.4 3 3 8 4 9.6
4 10 0 1 18.0 5 10 4 1 13.6 6 10 8 1 8.8
4 10 0 2 17.4 5 10 4 2 10.6 6 10 8 2 8.2
4 10 0 3 15.1 5 10 4 3 11.8 6 10 8 3 11.3
4 10 0 4 14.4 5 10 4 4 13.3 6 10 8 4 11.2
proc GLM;
  class DR D R block;
  model number= block DR;
*           D 3 3 3 10 10 10 ;
*           R 0 4 8 0 4 8 ;
*           DR 1 2 3 4 5 6 ;
  contrast 'D' DR 1 1 1 -1 -1 -1;
  contrast 'R lineal' DR -1 0 1 -1 0 1;
  contrast 'R quadratic' DR 1 -2 1 1 -2 1;
  contrast 'Int lineal R * D' DR -1 0 1 1 0 -1;
  contrast 'Int quadr. R * D' DR 1 -2 1 -1 2 -1;
run; quit;

```

Int lineal R*D: is $DR_3 - DR_1 = DR_6 - DR_4 \Rightarrow -1 \ 0 \ 1 \ 1 \ 0 \ -1$

Comparison Factorial opened as an RCBD vs. Factorial

Model number = TRT block; (factorial open as RCBD)

Class Level Information

Class	Levels	Values
TRT	6	1 2 3 4 5 6
block	4	1 2 3 4

Dependent Variable: number

Source	DF	SS	MS	F Value	Pr > F
Model	8	156.235	19.529	7.44	0.0005
Error	15	39.383	2.626		
Corr. Total	23	195.618			

Source	DF	SS	MS	F Value	Pr > F
block	3	0.582	0.194	0.07	0.9731
TRT	5	155.653	31.131	11.86	<.0001

Contrast	DF	Contrast SS	MS	F Value	Pr > F
D	1	1.500	1.500	0.57	0.4614
R lineal	1	152.522	152.522	58.09	<.0001
R quadratic	1	1.141	1.141	0.43	0.5198
Int R L*D	1	0.123	0.122	0.05	0.8319
Int R Q*D	1	0.367	0.367	0.14	0.7135

Previous analysis as a Factorial

Model number= D R D*R block;

Class Level Information

Class	Levels	Values
D	2	1 2
R	3	1 2 3
block	4	1 2 3 4

Source	DF	SS	MS	F Value	Pr > F
BLOCK	3	0.582	0.194	0.07	0.9731
D	1	1.500	1.500	0.57	0.4614
R	2	153.663	76.832	29.26	0.0001
D*R	2	0.490	0.245	0.09	0.9114

Contrast	DF	Contrast SS	MS	F Value	Pr > F
R lineal	1	152.522	152.522	58.09	0.0001
R quadratic	1	1.141	1.141	0.43	0.5198