The mean and variance of linear functions

1) Add a constant C to each observation

$\sum (Y+C) = nC + \sum Y$

Example: Add 10 to each observation

(3, 5) Mean 4, $s^2 = 2$

(13, 15) Mean 14, $s^2 = 2$

Mean of the new variable = Previous mean + 10

s^2 new variable = Previous s^2

2) Multiply each observation by a constant

$\sum (Y*C) = C * \sum Y$

Example: Multiply each observation * 10

(3, 5) Mean 4, $s^2 = 2$

(30, 50) Mean 40, $s^2 = 200$

Mean of the new variable = Previous mean * 10

s^2 of the new variable = Previous s^2 * 100

3) Add two independent random variables

$Z = X_1 + X_2$

$X_1 \sim N(\mu_1, \sigma^2_1)$ and $X_2 \sim N(\mu_2, \sigma^2_2) \Rightarrow X_1 + X_2 \sim N(\mu_1+\mu_2, \sigma^2_1 + \sigma^2_2)$

Mean of Z = Mean of X_1 + mean of X_2

Variance of Z = $s^2_1 + s^2_2$

4) Subtract two independent random variables

$Z = X_1 - X_2$

$X_1 \sim N(\mu_1, \sigma^2_1)$ and $X_2 \sim N(\mu_2, \sigma^2_2) \Rightarrow X_1 - X_2 \sim N(\mu_1-\mu_2, \sigma^2_1 + \sigma^2_2)$

Mean of Z = Mean of X_1 - mean of X_2

Variance of Z = $s^2_1 + s^2_2$